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| January 18, 2015 | Matt Landreman |

# Effect on fluxes of the poloidally varying electrostatic potential

In these notes, we point out a term in the heat flux which is not computed by many neoclassical codes, including DKES, momentum-corrected DKES, or versions of SFINCS prior to 3. This term appears formally as large as the part of the heat flux which is usually computed, even when the species charge is Z=1. Computing this term requires knowledge of the variation of the electrostatic potential  on a flux surface.

Suppose we have a solution  to the linear drift-kinetic equation in which there is no parallel variation of the electrostatic potential, as in DKES:

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Here,



is a stationary Maxwellian,  denotes species, and  is the magnetic drift (not including any  drift). We neglect the radial electric field in for simplicity. The solution  gives rise to a certain particle flux



and an energy flux

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(The subscript  on the fluxes - indicates these fluxes are associated with the radial *magnetic* drift.)

Now consider the “real” drift-kinetic equation, in which parallel variation of the electrostatic potential  is retained. We denote the solution to this form of the kinetic equation by :

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The solution to can be written in terms of the solution to :

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If we evaluate the radial particle and energy fluxes associated with  caused by just the magnetic drifts (ignoring the radial  drift,) we do not get the same fluxes we got before:



and



In other words, including the blue  term in causes the radial fluxes to change by the red terms in -. Using

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we can evaluate the  integrals in the red terms of -, giving



and

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Using the MHD equilibrium relation , along with

,

then - can be integrated by parts to find



and

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When  varies on a flux surface, there will also be a radial  drift. The particle and heat fluxes associated with this drift are, considering just  as opposed to ,



and

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Combining -, we find



and

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The left-hand sides of - are the “correct” fluxes, since they account for variation of  on a flux surface, whereas  and  do not.

Eq shows that ignoring  in the kinetic equation does not change the particle flux: we get the same particle flux if we do include  and then account for the extra radial flux from the radial  drift. However, shows that the heat flux *does* change when  is included in the kinetic equation, and this change cannot be compensated by accounting for the radial heat flux from the radial  drift. Since the red term in cannot be computed without knowledge of the variation of  on a flux surface, it appears that the “true” radial heat flux  cannot be computed from codes that simply report the quantity , such as DKES, momentum-corrected DKES, or versions of SFINCS prior to 3.

Using the standard estimates  (where  is the part of  which varies on a flux surface), , , and , where  and  is a macroscopic scale, we find

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Thus, the red term which is not computed by the aforementioned codes is formally as large in the  expansion as the term  which is usually computed.